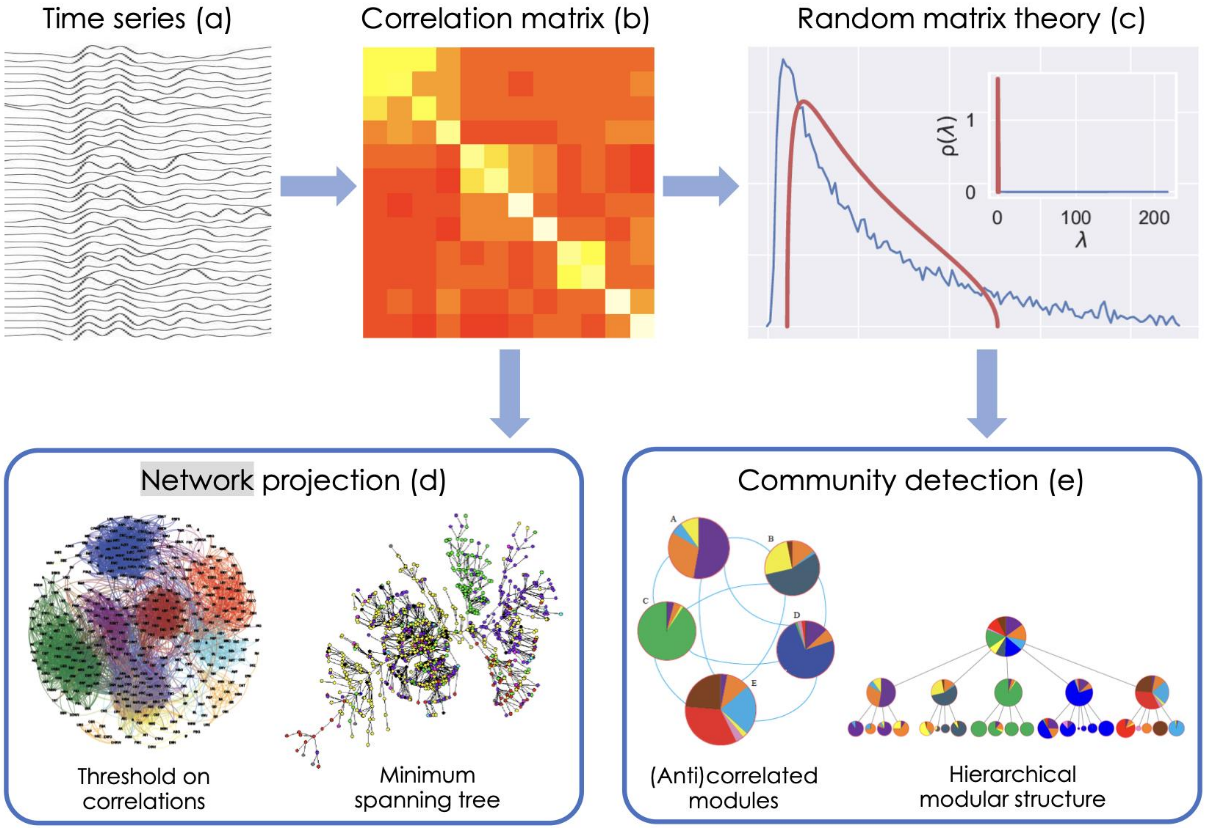
The Physics of Financial Networks

What do influencers and large banks have in common?

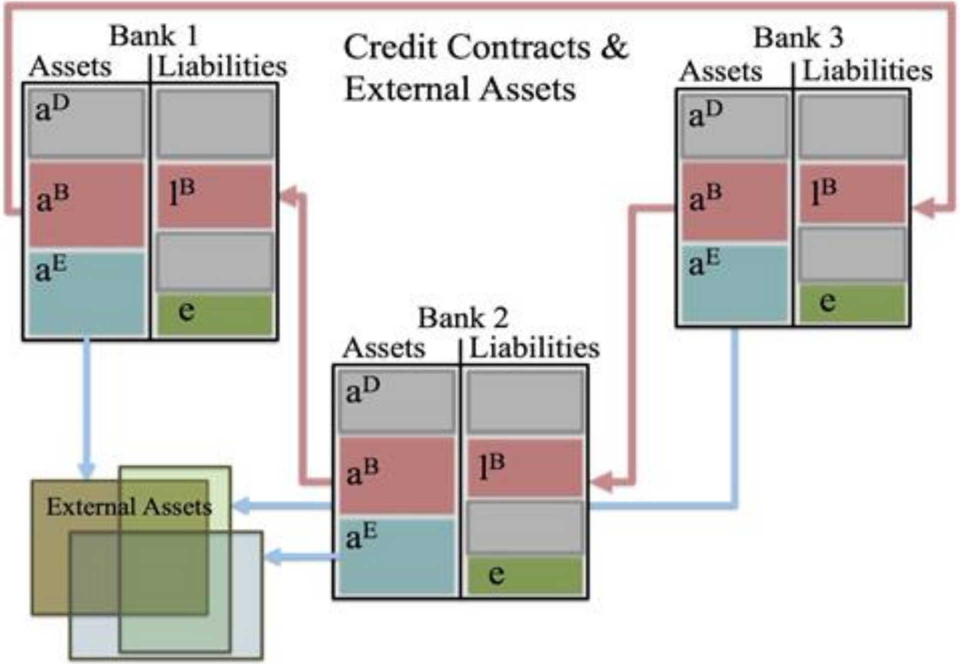
Networks

Economic agents (individuals, firms, banks) enter legal, transactional, financial relationships and these relationships constitute a complex multi-layered weighted temporal structure of the economy

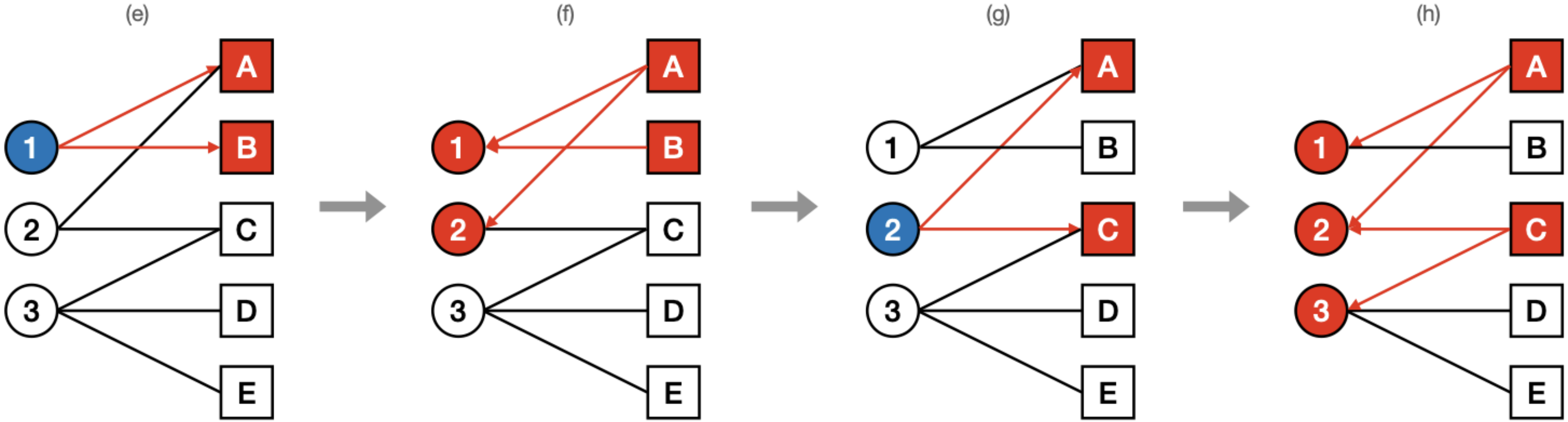
Why financial networks?



Why financial networks?



Why financial networks?



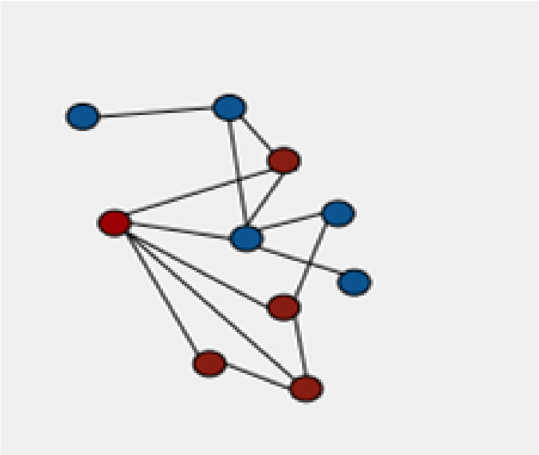
Why financial networks?

The evolution of the variables associated with the economic institutions on these networks are **influenced by** and **influence** the network of connections, creating a feedback dynamics between the networks and the economic agents.

Therefore, we investigate both:

- the dynamics of the agents **on financial networks**
- the dynamics **of financial networks**

Models of static networks



Fitness model

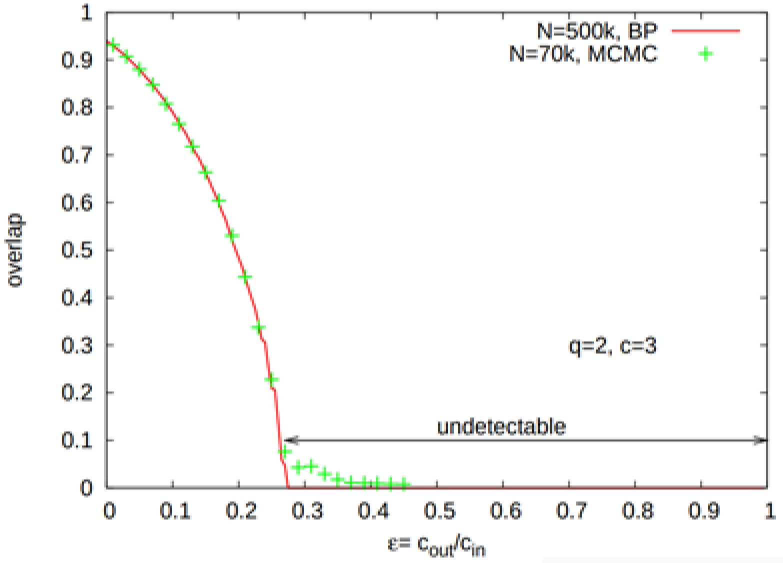
$$\mathbb{P}(\boldsymbol{A}^t|\Theta^t) = \prod_{i,j>i} \mathbb{P}(A^t_{ij}|\theta^t_i, \theta^t_j) = \prod_{i,j>i} \frac{e^{A^t_{ij}(\theta^t_i+\theta^t_j)}}{1+e^{(\theta^t_i+\theta^t_j)}}$$

- Models fitness parameters per each node
- It defines an affinity matrix, i.e. a link probability based on the respectives fitness values
- Useless for link prediction without a model for the dynamics

Stochastic block model graphs

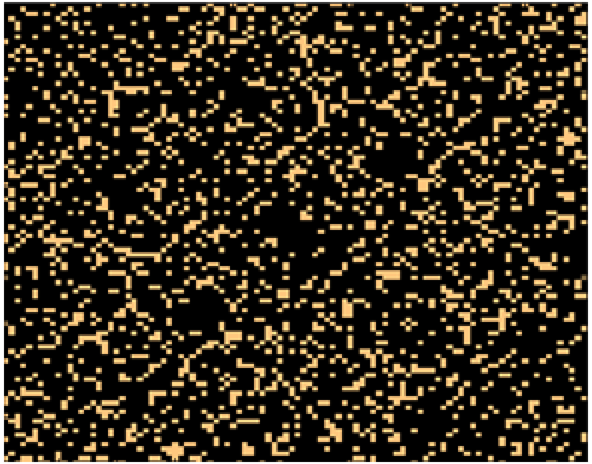
A **stochastic block model graph** is a random graph where:

- each node is assigned to a block g_i
- each unordered (undirected case) pair of different nodes is randomly linked with probability p_{ab}, where $a=g_i$ and $b=g_j$

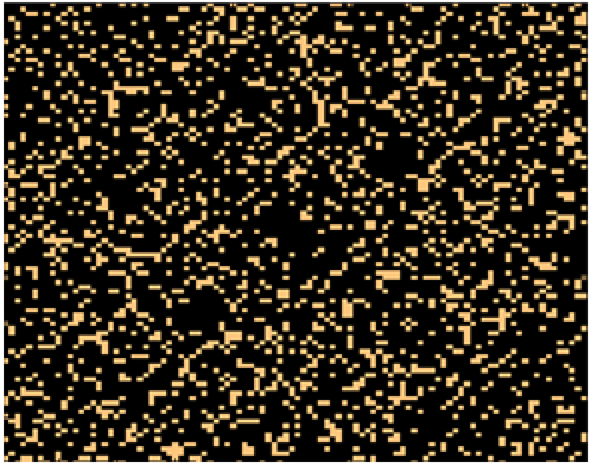


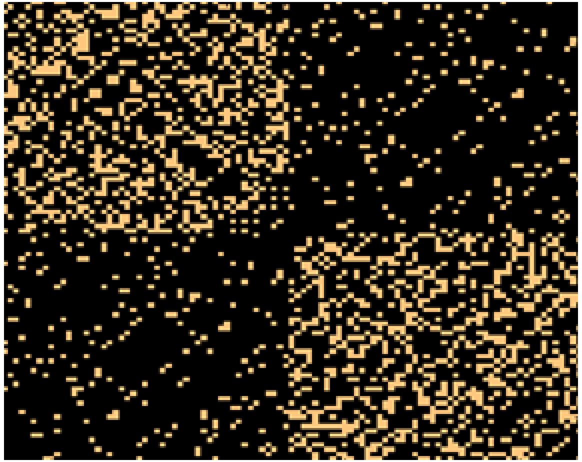
Decelle et al.

Community detection



Community detection





Identifying core-periphery structure

Research question:

- What is the core of a financial network and how do we find it?

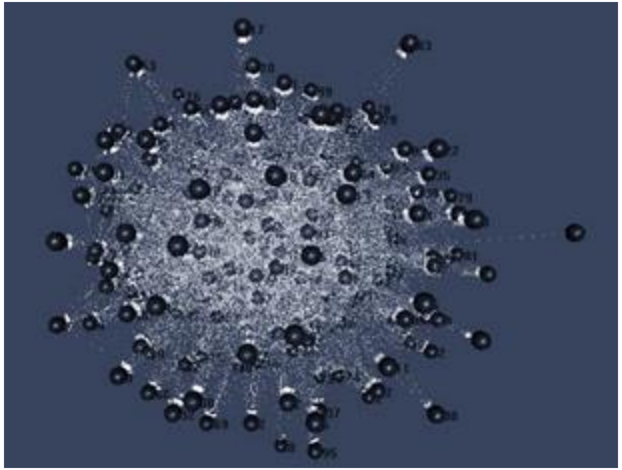


Figure taken from Fricke and Lux (2012)

Relevance and policy implications:

- Risk management for banks in the core and in the periphery can be different
- Network structure affects systemic risk, core banks can be more vulnerable and systemic
- A given position in the structure may correspond to different business strategies
- Capital requirements for core banks can be different

- Fricke, Daniel, Lux, Thomas (2012) : Core-periphery structure in the overnight money market: Evidence from the e-MID trading platform, *Kiel Working Paper*, No. 1759
- Csermely, P., London, A., Wu, L. Y., & Uzzi, B. (2013). Structure and dynamics of core/periphery networks. *Journal of Complex Networks*, *1*(2), 93-123.
- in ’t Veld D, van Lelyveld I (2014) Finding the core: network structure in interbank markets. *J Bank Financ* 49:27–40

Identifying strategies in interbank networks

Research question:

- How do we know if there is a core-periphery structure in a complex network, e.g. a network of financial transactions?

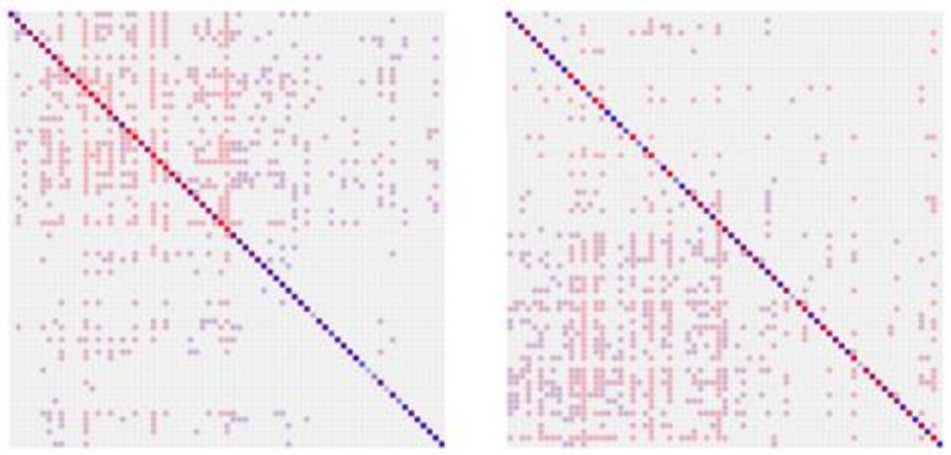


Figure taken from Barucca and Lillo (2016) based on granular eMID data

Methodology:

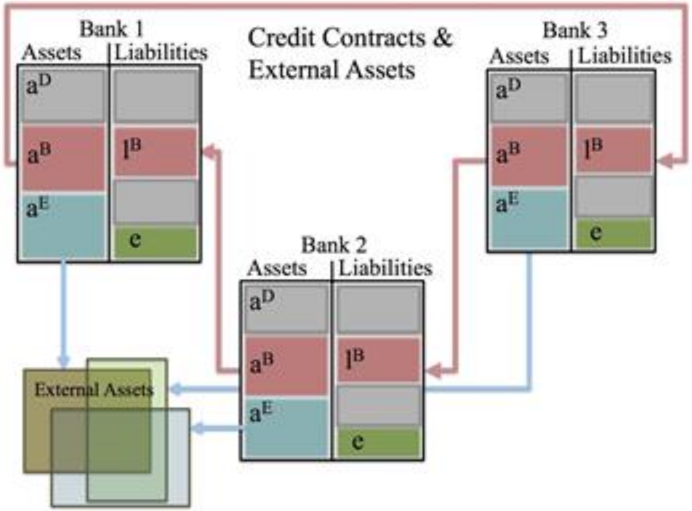
- Stochastic block model generating networks with an arbitrary block structure
- Expectation-Maximization until convergence to the most probable assignment of nodes into groups

Network valuation in financial systems

Research question:

- What is the value of an interbank asset embedded in a network of liabilities?

Network valuation in financial systems, PB, M. Bardoscia, M. D'Errico, G. Visentin, F. Caccioli, G. Caldarelli, S. Battiston (To be submitted to *Mathematical Finance*)



$$\tilde{E}_{it} = \mathbb{V}_i^{(e)}(A_{it}^{(e)}|\tilde{E}_{it}) - L_i^{(e)} + \sum_j \mathbb{V}(A_{ij}|\tilde{E}_{jt},\ldots) - \sum_j L_{ij}$$

$$\frac{\mathbb{V}(A_{ij}|\tilde{E}_{jt},\ldots)}{A_{ij}} = \mathbb{E}[\mathbb{1}_{\tilde{E}_{jT}>0} + R(\frac{\tilde{E}_{jT}+\bar{p}_j}{\bar{p}_j})^{+}\mathbb{1}_{\tilde{E}_{jT}\leq 0}]_{\Delta A_j^{(e)}}$$

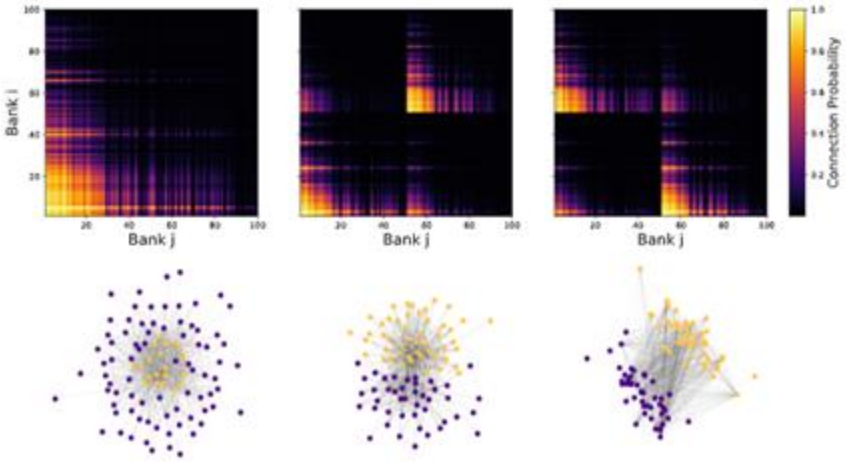
Paolo Barucca – UCL

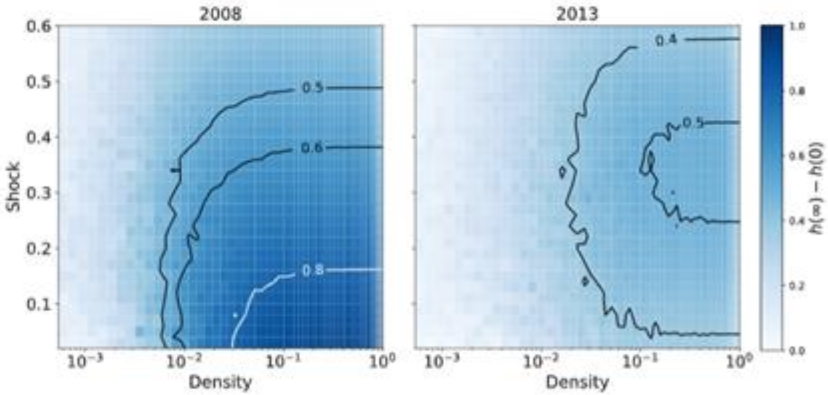
Network sensitivity of systemic risk

Ramadiah, Amanah, Domenico Di Gangi, D. Sardo, Valentina Macchiati, Minh Tuan Pham, Francesco Pinotti, Mateusz Wiliński, Paolo Barucca, and Giulio Cimini. "Network sensitivity of systemic risk." *Journal of Network Theory in Finance* 5, no. 3 (2019).

Research question:

- How is systemic risk depending on the underlying financial networks?

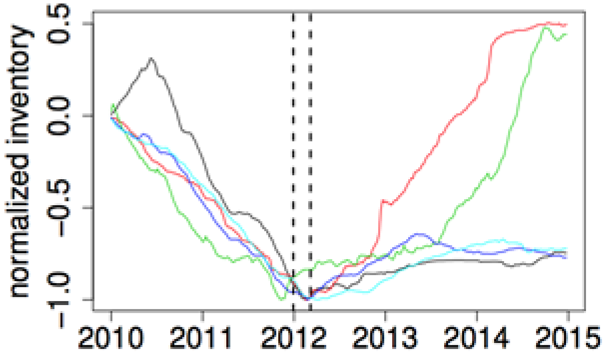




Identifying strategies in interbank networks

Main findings:

- Degree heterogeneity can create a (spurious) core-periphery structure
- Time aggregation leads to degree heterogeneity and to the emergence of a core-periphery structure
- eMID mainly displays a ***bipartite*** structure on a daily and weekly basis with a main division in lenders and borrowers
- Following the ECB LTRO measures the behaviour of banks in the eMID market abruptly changed



References:

- Barucca, P., Lillo, F. (2016). Disentangling bipartite and core-periphery structure in financial networks. *Chaos, Solitons & Fractals*, *88*, 244-253.

- Barucca, P., Lillo, F. (2017). The organization of the interbank network and how ECB unconventional measures affected the e-MID overnight market. *Comput Manag Sci,* 1-22.

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Models of temporal networks I

Snapshot models

Models of temporal networks I

Snapshot models

- DAR(p) discrete autoregressive processes
- SSI single snapshot inference
- TGRG temporally generalized random graphs
- DAR-TGRG discrete autoregressive temporally generalized random graphs
- Non-linear regression models with network features

The simplest model: DAR(1)

$$A_{ij}^{t} = V_{ij}^{t} A_{ij}^{t-1} + (1 - V_{ij}^{t}) Y_{ij}^{t} \quad \forall i, j = 1, \ldots, N \text{ and } j > i$$

- A is the adjacency matrix
- V and Y are Bernoulli variables

Single snapshot inference (SSI)

$$\mathbb{P}(\boldsymbol{A}^t|\Theta^t) = \prod_{i,j>i} \mathbb{P}(A^t_{ij}|\theta^t_i, \theta^t_j) = \prod_{i,j>i} \frac{e^{A^t_{ij}(\theta^t_i+\theta^t_j)}}{1+e^{(\theta^t_i+\theta^t_j)}}$$

- No autoregressive model of fitness parameters
- No link copying mechanism
- No memory in the model
- Useless for link prediction without a model for the fitness parameters

Temporally generalized random graphs (TGRG)

$$\theta_i^t = \phi_{0,i} + \phi_{1,i}\theta_i^{t-1} + \epsilon_i^t\,, \quad \forall i = 1, ..., N$$

$$\begin{cases} \mathbb{P}(\theta_i^t|\theta_i^{t-1}, \boldsymbol{\Phi}_i) & = f(\theta_i^t|\phi_{0,i} + \phi_{1,i}\theta_i^{t-1}, \sigma_i^2) \;\; \forall i = 1, ..., N \\ \mathbb{P}(\boldsymbol{A}^t|\Theta^t) & = \prod_{i,j>i} \mathbb{P}(A_{ij}^t|\theta_i^t, \theta_j^t) = \prod_{i,j>i} \frac{e^{A_{ij}^t(\theta_i^t+\theta_j^t)}}{1+e^{(\theta_i^t+\theta_j^t)}} \end{cases}$$

- AR(1) model for the fitness parameters
- No link copying mechanism
- The memory is only in the fitness parameters

DAR-TGRG

$$\begin{cases} \mathbb{P}(\theta_i^t|\theta_i^{t-1}, \mathbf{\Phi}_i) & = f(\theta_i^t|\phi_{0,i} + \phi_{1,i}\theta_i^{t-1}, \sigma_i^2) \quad \forall i = 1, ..., N \\ \mathbb{P}(\boldsymbol{A}^t|\boldsymbol{A}^{t-1}, \Theta^t, \boldsymbol{\alpha}) & = \prod_{i,j>i} \left(\alpha_{ij} \mathbb{1}_{A_{ij}^t A_{ij}^{t-1}} + (1 - \alpha_{ij}) \frac{e^{A_{ij}^t(\theta_i^t + \theta_j^t)}}{1+e^{(\theta_i^t+\theta_j^t)}} \right) \end{cases}$$

- AR(1) model for the fitness parameters
- Link copying mechanism
- The memory is both in the fitness parameters and in the sampled matrices

Recognising preferential trading and predicting links

Research question:

- Can we provide an analytic model of dynamic networks that would account for their persistence?
- Can we calibrate such model efficiently and predict future links, i.e. future transactions in the case of an interbank market?

Methodology:

- Fitness model generating networks with an arbitrary degree distribution
- Markov Process for the latent variables and for individual links

$$\begin{cases} \mathbb{P}(\theta_i^t | \theta_i^{t-1}, \mathbf{\Phi}_i) & = f(\theta_i^t | \phi_{0,i} + \phi_{1,i}\theta_i^{t-1}, \sigma_i^2) \quad \forall i = 1, ..., N \\ \mathbb{P}(\boldsymbol{A}^t | \boldsymbol{A}^{t-1}, \Theta^t, \boldsymbol{\alpha}) & = \prod_{i,j>i} \left(\alpha_{ij} \mathbb{I}_{A_{ij}^t A_{ij}^{t-1}} + (1 - \alpha_{ij}) \frac{e^{A_{ij}^t (\theta_i^t + \theta_j^t)}}{1 + e^{(\theta_i^t + \theta_j^t)}} \right) \end{cases}$$

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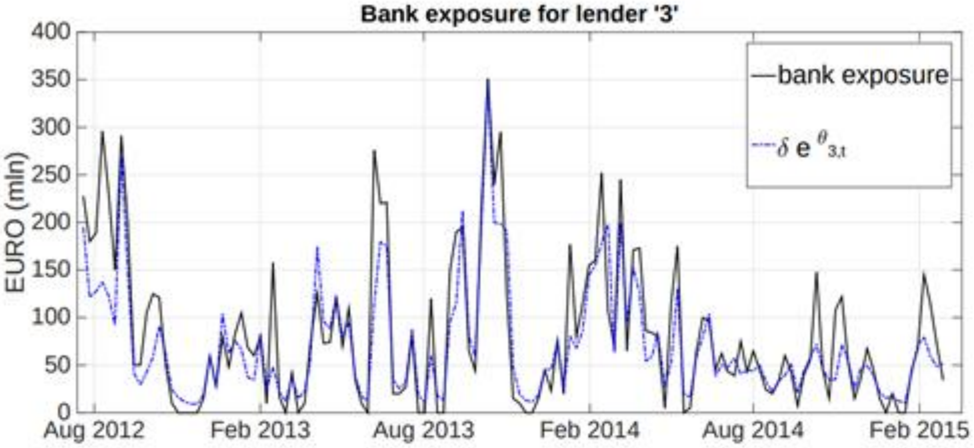
Time-varying fitness

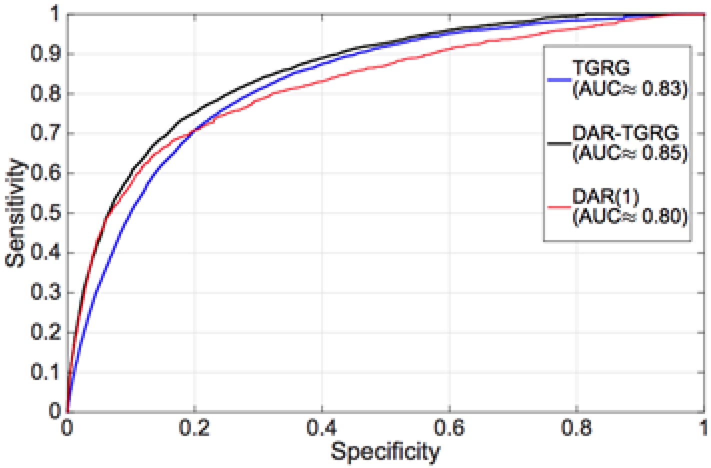


An example of time-varying fitness compared with the bank exposure for a given bank

Recognising preferential trading and predicting links

Main findings:

- The empirical results allow us to recognise preferential lending in the interbank market,
- A method that does not account for time-varying network topologies tends to overestimate preferential linkage.



True positive rate (sensitivity) vs true negative rate (specificity) in link prediction

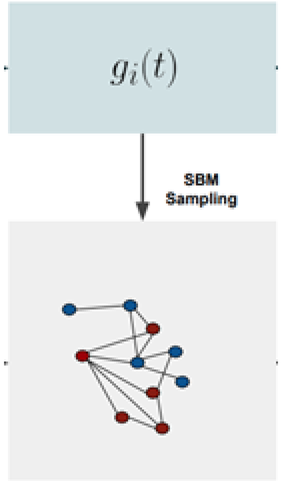
Reference:

Mazzarisi, Piero, Paolo Barucca, Fabrizio Lillo, and Daniele Tantari. "A dynamic network model with persistent links and node-specific latent variables, with an application to the interbank market." *European Journal of Operational Research* 281, no. 1 (2020): 50-65.

Following the evolution of communities

Research question:

- Can we follow the community structure of a dynamic network through time?
- Is there a detectability threshold for dynamic networks?



Methodology:

- Dynamic Stochastic Block Model generating networks with an arbitrary block structure
- Markov Processes both for the community structure and for individual links

Following the evolution of communities

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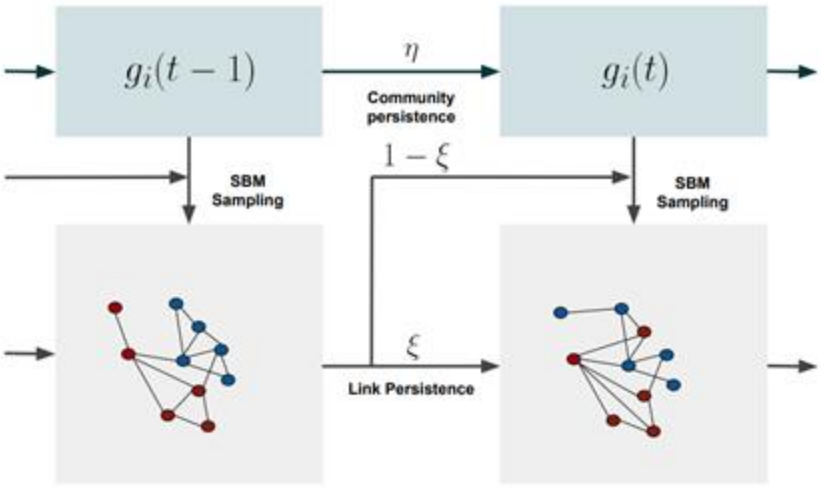


Methodology:

- Dynamic Stochastic Block Model generating networks with an arbitrary block structure
- Markov Processes both for the community structure and for individual links

Persistent DSBM

A **persistent DSBM** is an ensemble of dynamic networks where:

- at each time step, each node is assigned to a block g_i with a probability depending on the previous assignments

- at each time step each pair of different nodes either copies its previous relationship or links with probability p_{ab}, where $a=g_i$ and $b=g_j$

Following the evolution of communities

Research question:

- Can we follow the community structure of a dynamic network through time?
- Is there a detectability threshold for dynamic networks?

$$P(\boldsymbol{g}) = \prod_{i=1}^{N} \left[\prod_{t=1}^{T} \eta \, \delta_{g_i^t, g_i^{(t-1)}} + (1-\eta) q_{g_i^t} \right] q_{g_i^0}$$

$$\begin{aligned} P(\boldsymbol{A}|\boldsymbol{g}) \quad &= \quad \prod_{(ij)}^{N} p_{g_i^0 g_j^0}^{A_{ij}^0} (1 - p_{g_i^0 g_j^0})^{1-A_{ij}^0} \times \\ &\times \quad \prod_{t=1}^{T} \left(\xi \, \delta_{A_{ij}^t, A_{ij}^{(t-1)}} + (1-\xi) p_{g_i^t g_j^t}^{A_{ij}^t} (1 - p_{g_i^t g_j^t})^{1-A_{ij}^t} \right), \end{aligned}$$

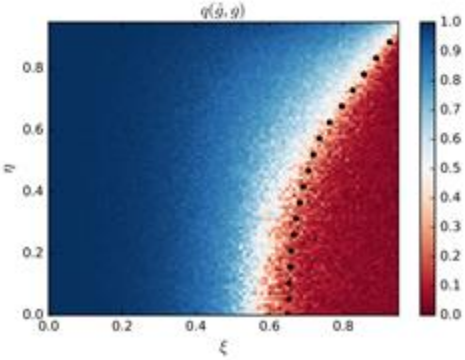
Methodology:

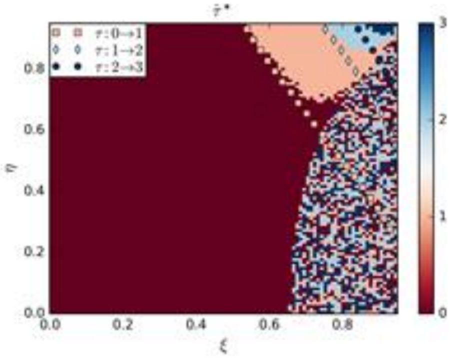
- Dynamic Stochastic Block Model generating networks with an arbitrary block structure
- Markov Processes both for the community structure and for individual links

Following the evolution of communities

Main findings:

- Time-lagged inference: the identification of past communities rather than present ones,
- We propose a corrected algorithm, the Lagged Snapshot Dynamic algorithm, for community detection in dynamic networks,
- We analytically and numerically characterize the detectability transitions of such algorithm as a function of the memory parameters of the model, yielding the known static result in case of vanishing memory parameters.





References:

- Barucca, P., Lillo, F., Mazzarisi, P., and Tantari, D. (2017). Disentangling group and link persistence in dynamic stochastic block models. *arXiv preprint arXiv:1701.05804*.

Structural edge importance in capital markets transactions

Main findings:

- We define a structural importance metric, le, for the edges of a network.
- The metric is based on perturbing the adjacency matrix and observing the change in its largest eigenvalues.
- We propose a model of network evolution where this metric controls the probabilities of subsequent edge changes.

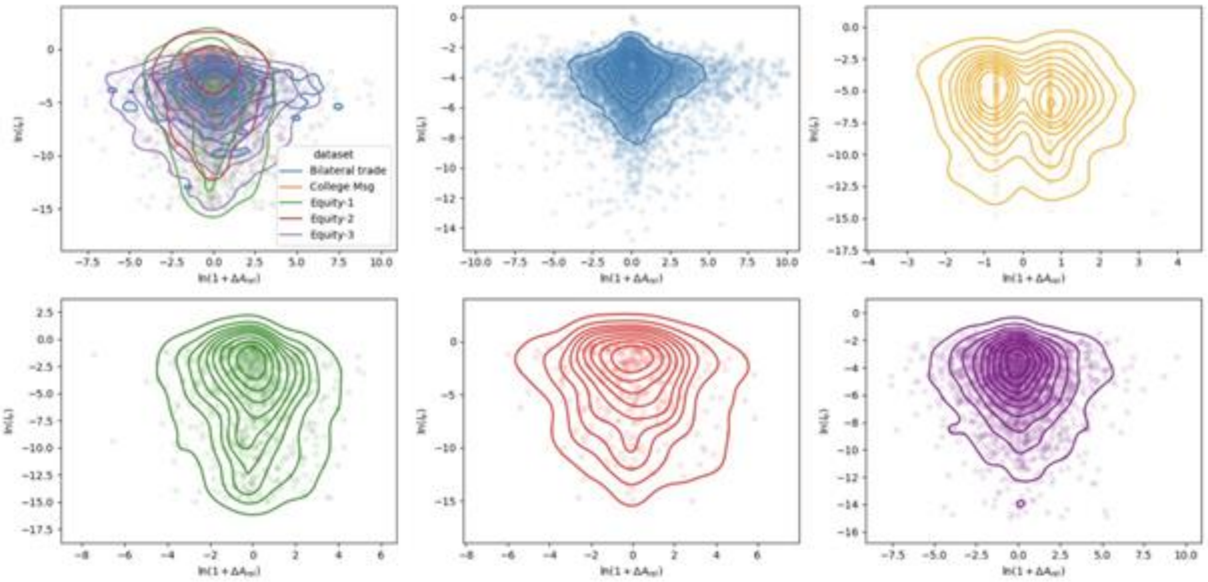
$$l_e = \frac{\partial \lambda}{\partial A_{ij}} \qquad A_{ij}^{t+1} = \mathcal{V}_{ij}^t A_{ij}^t \mathcal{U}_{ij}^t + (1 - \mathcal{V}_{ij}^t) A_{ij}^t$$

Reference:

Seabrook, Isobel E., Paolo Barucca, and Fabio Caccioli. "Evaluating structural edge importance in temporal networks." *EPJ Data Science* 10, no. 1 (2021): 23.

Structural edge importance in capital markets transactions

We show using synthetic data how the parameters of the model are related to the capability of predicting whether an edge will change from its value of le.



Structural edge importance in capital markets transactions

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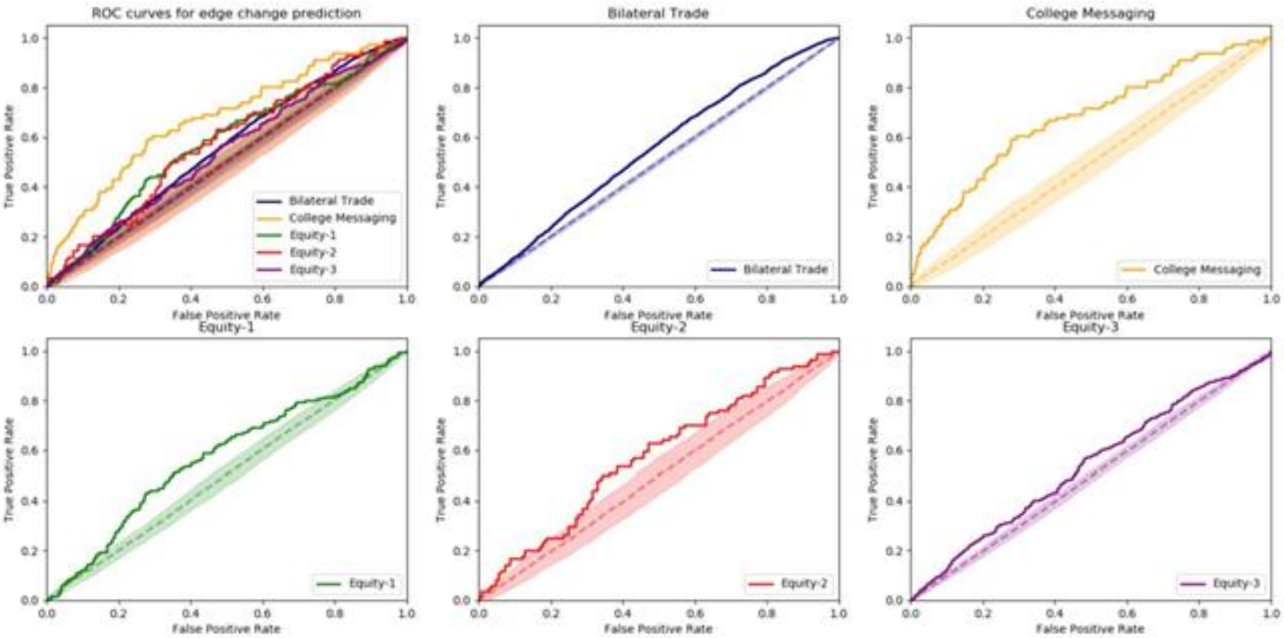
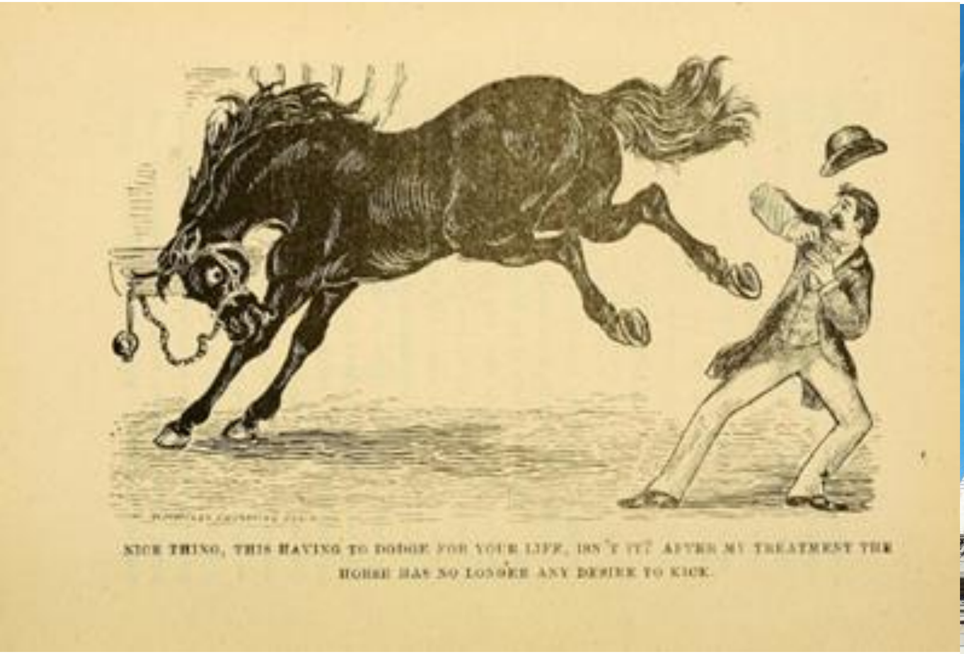


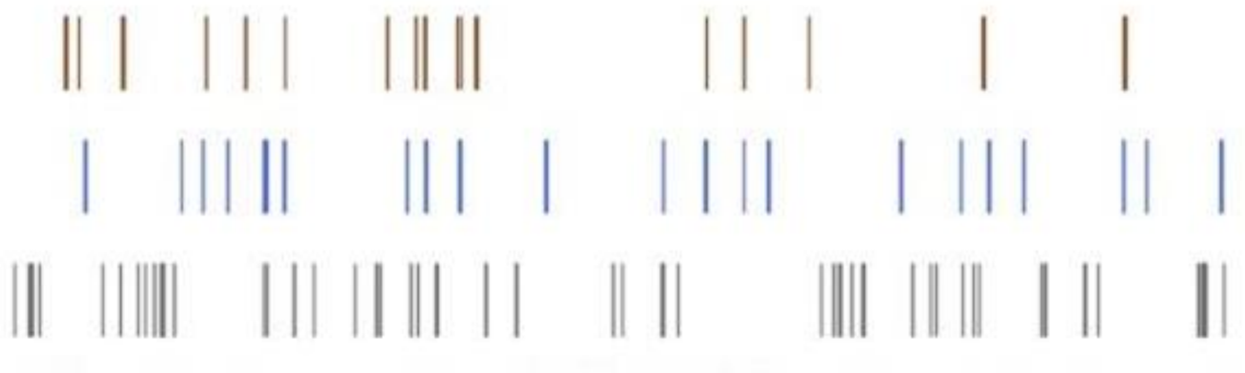
What do stocks and horses have in common?



Ilustration of The von Bortkeiwicz Prussian horse-kicking survey / The Bank of England

Models of temporal networks II

Timestamped models



Hawkes processes for bursty transactions

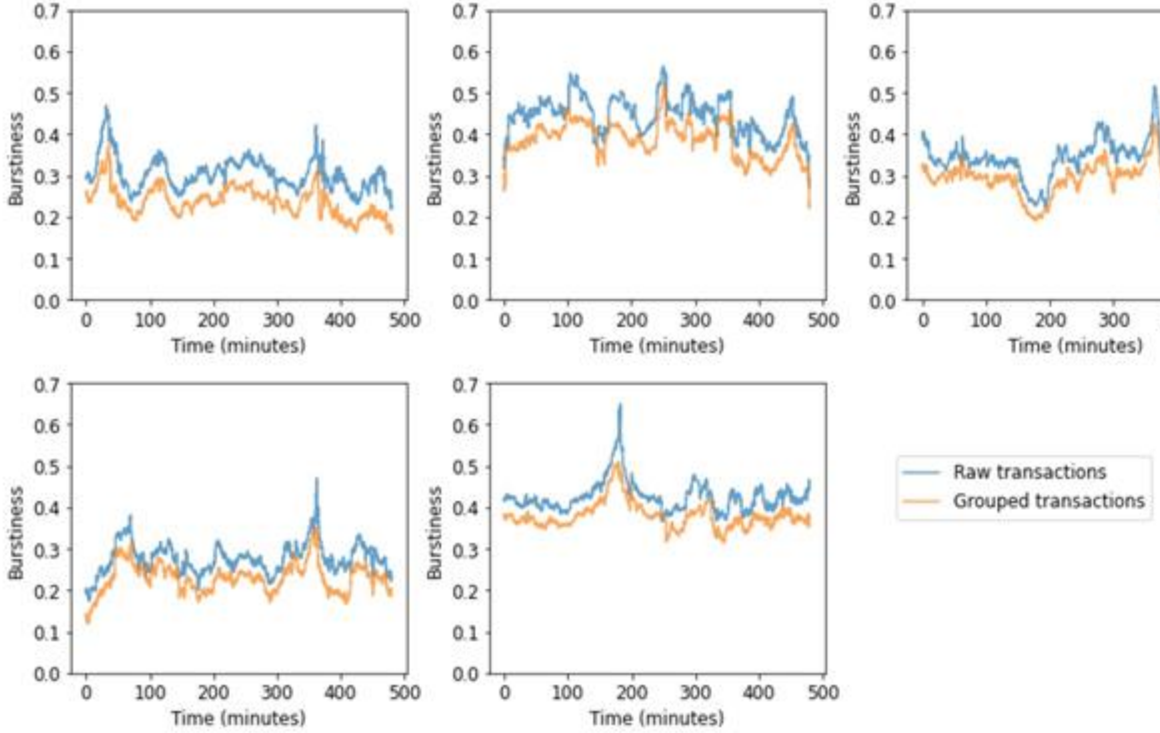
Main findings:

- we use transaction reports for five FTSE 100 stocks,
- we fit Hawkes processes at the overall transaction level, at the level of individual counterparty relationships, and for trades executed via central clearing counterparties
- we generate synthetic transaction sequences which display similar properties to real transaction sequences.
- we quantify burstiness depending on the number of hubs (CCPs) in the transaction networks

Reference:

Seabrook, Isobel E., Paolo Barucca, and Fabio Caccioli. "Modeling equity transaction networks as bursty processes" *Working Paper*

Hawkes processes for bursty transactions



Reference:

Seabrook, Isobel E., Paolo Barucca, and Fabio Caccioli. "Modeling equity transaction networks as bursty processes" *Working Paper*

Hawkes processes for bursty transactions

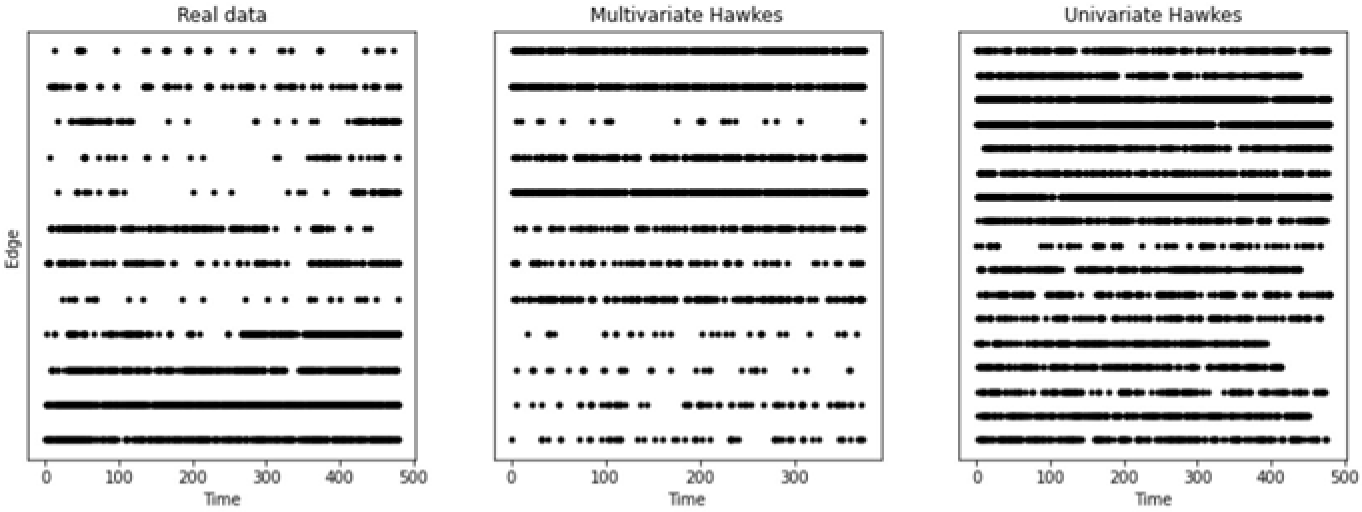
$$\lambda(t) = \mu + \sum_{t_k < t} \phi(t - t_k)$$

$$\mathcal{L} = \log \frac{\prod_{i=1}^{n} \lambda(t_i)}{\exp \int_0^T \lambda(t)dt} = \sum_{i=1}^{n} \log \lambda(t_i) - \int_0^T \lambda(t)dt$$

Reference:

Seabrook, Isobel E., Paolo Barucca, and Fabio Caccioli. "Modeling equity transaction networks as bursty processes" *Working Paper*

Hawkes processes for bursty transactions



Reference:

Seabrook, Isobel E., Paolo Barucca, and Fabio Caccioli. "Modeling equity transaction networks as bursty processes" *Working Paper*

Maximum Entropy Temporal Networks

- Can we find a theoretically founded model to introduce constraints in temporal networks which reproduce bursty patterns, node-node, and edge-edge correlations?
- Can we find a simple unified picture for detecting statistically significant patterns of temporal edges in time-varying networked systems?

Maximum Entropy Temporal Networks

$$
\begin{aligned}
\mathcal{L} = & - \sum_{(i,j)\in\mathcal{S}} \int_0^T \left(\lambda_{ij} \log \lambda_{ij} - \lambda_{ij}\right) dt \\
& + \sum_{r=1}^{R} \int_0^T \alpha_r(t) \Big(\sum_{(i,j)\in E_r \cap \mathcal{S}} \lambda_{ij}(t) - G_r(t) \Big) dt \\
& + \sum_{(i,j)\in\mathcal{S}} \Theta_{ij} \Big(\int_0^T \lambda_{ij}\, dt - \mu_{ij}^{\star} \Big) + \sum_i \theta_i^{\mathrm{out}} \Big(\sum_j \int_0^T \lambda_{ij}\, dt - s_i^{\mathrm{out},\star} \Big) \\
& + \sum_j \theta_j^{\mathrm{in}} \Big(\sum_i \int_0^T \lambda_{ij}\, dt - s_j^{\mathrm{in},\star} \Big) + \sum_{a,b\in\mathcal{B}} \zeta_{ab} \Big(\sum_{i\in a} \sum_{j\in b} \int_0^T \lambda_{ij}\, dt - C_{ab}^{\star} \Big).
\end{aligned}
$$

The constraints

$$\frac{\delta \mathcal{L}}{\delta \lambda_{ij}(t)} = -\log \lambda_{ij}(t) - 1 + \alpha_{r(i,j)}(t) + \Psi_{ij} = 0,$$

- To enforce these constraints for the maximum entropy ensemble, we need to define the full intensity function a-priori, i.e. before the sampling of the edges: the paths need to be frozen.
- We can then compare the ensemble with another ensemble which is following a self-reinforcing dynamics, i.e. such that the intensities are history-dependent.

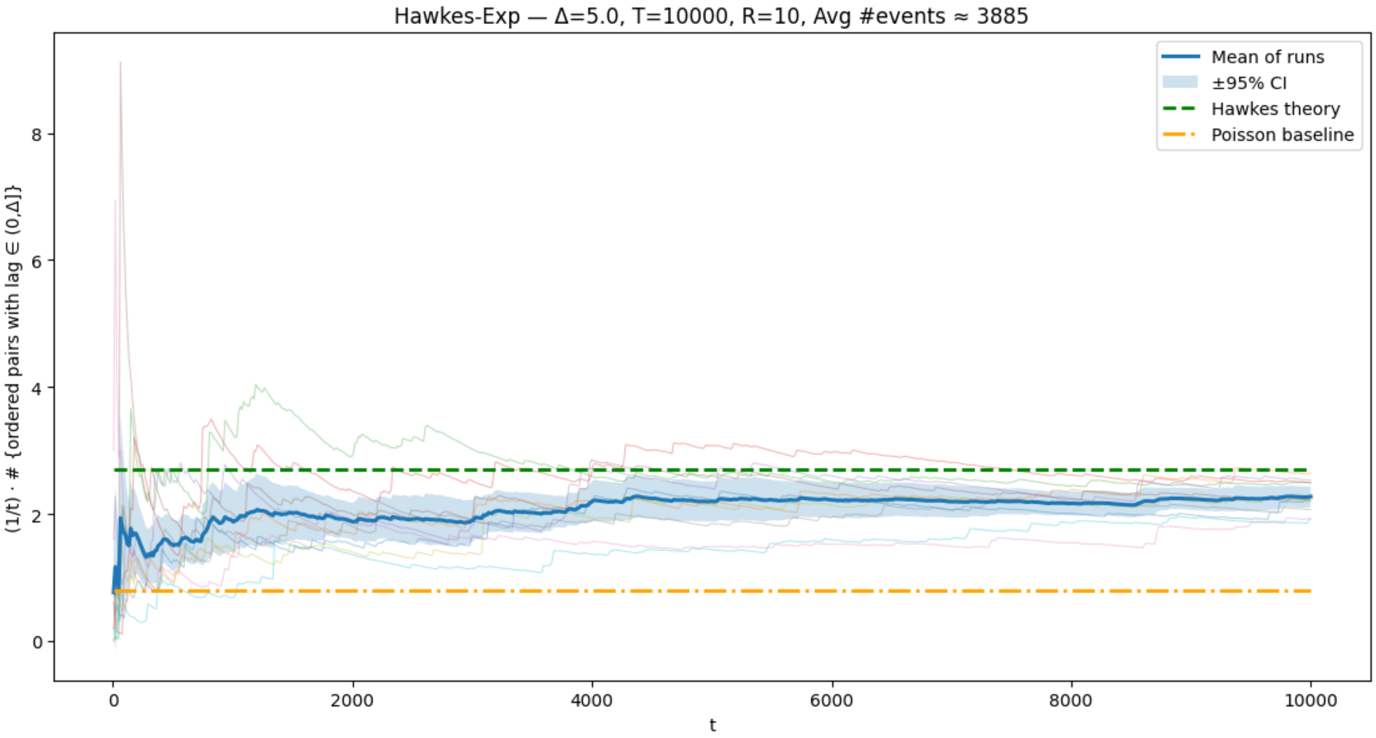
The constraints

$$\frac{\delta \mathcal{L}}{\delta \lambda_{ij}(t)} = -\log \lambda_{ij}(t) - 1 + \alpha_{r(i,j)}(t) + \Psi_{ij} = 0,$$

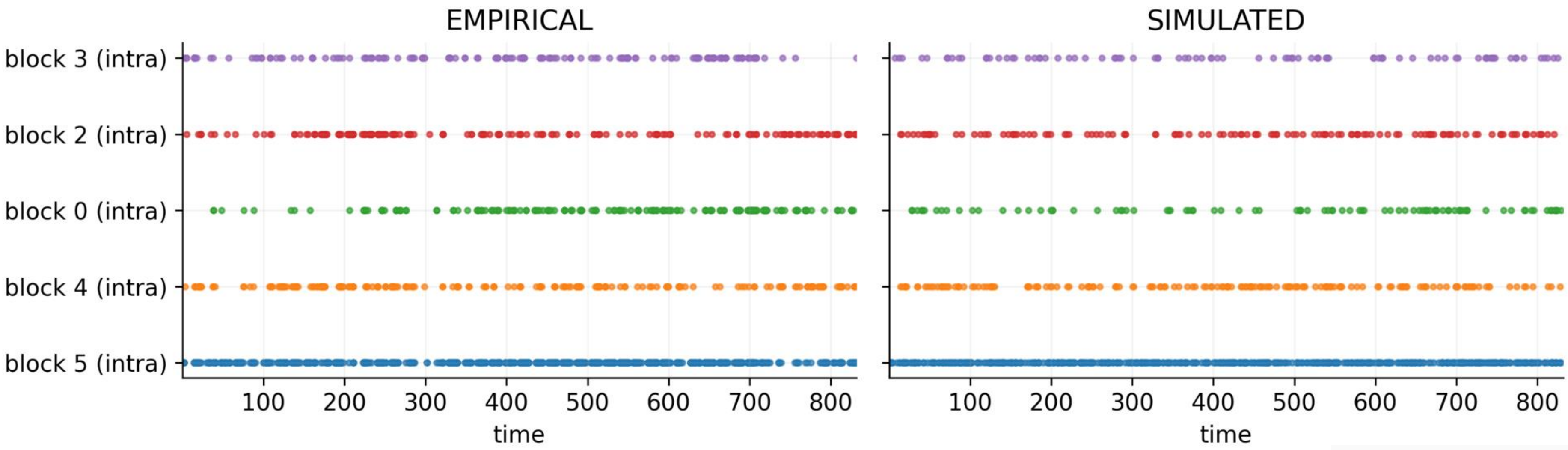
$$\text{Edge totals:} \quad \int_0^T \lambda_{ij}(t)\ \mathrm{d}t = \mu_{ij} \quad \Rightarrow \quad \Pi_{ij} = \mu_{ij}/\Lambda^{tot}.$$

$$\text{Strengths:} \quad \sum_{j \neq i} \Pi_{ij} = \frac{s_i^{\text{out}}}{\Lambda^{tot}}, \qquad \sum_{i \neq j} \Pi_{ij} = \frac{s_j^{\text{in}}}{\Lambda^{tot}},$$

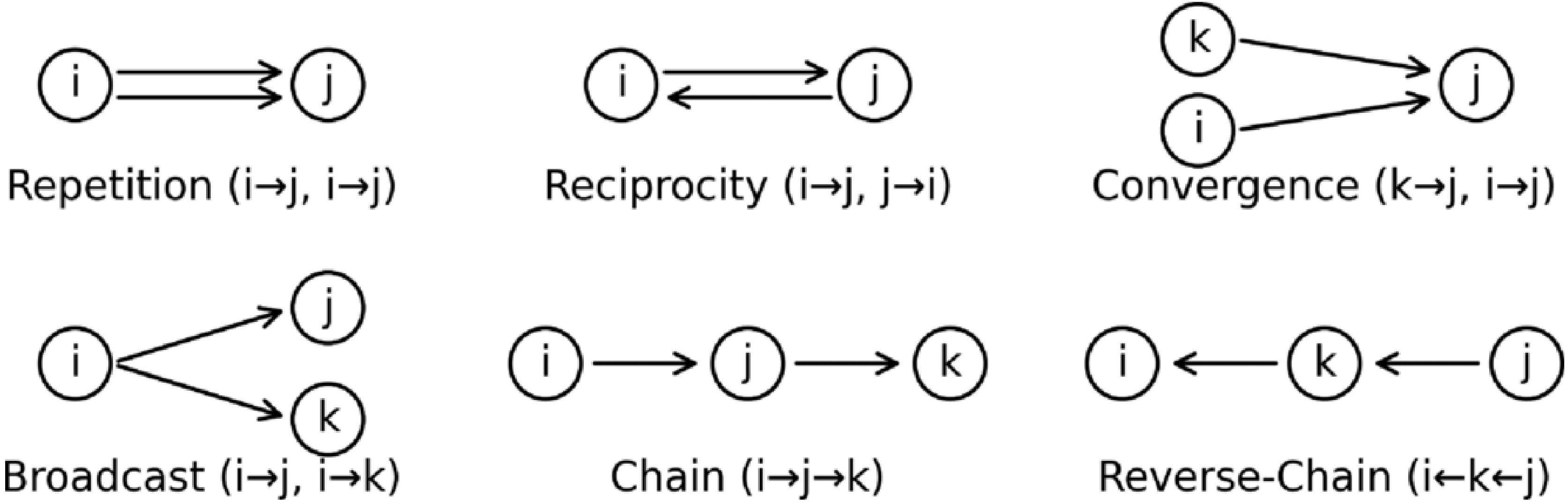
Testing frozen-path approximation



Simulating realistic temporal block patterns



Testing motif counts



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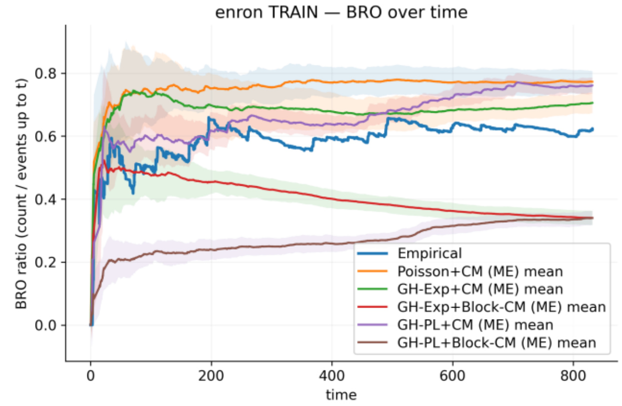

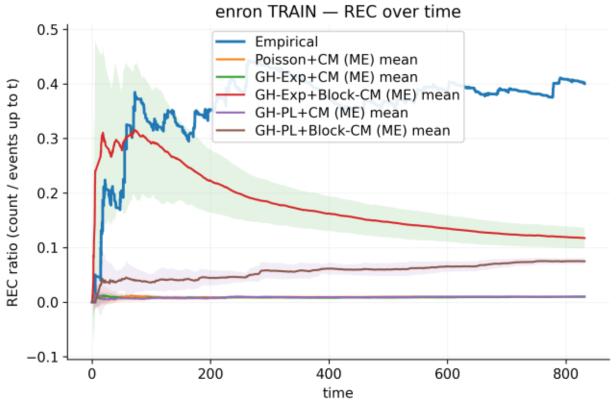

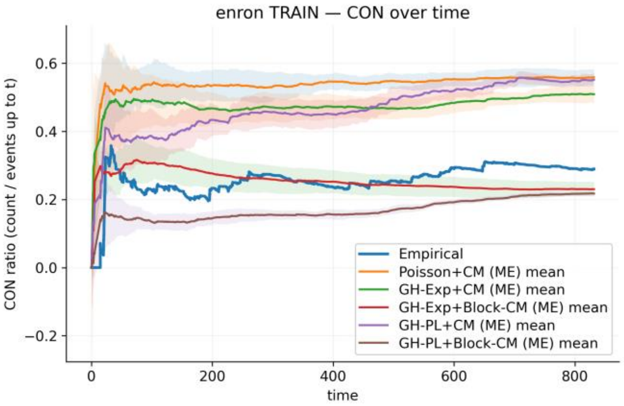

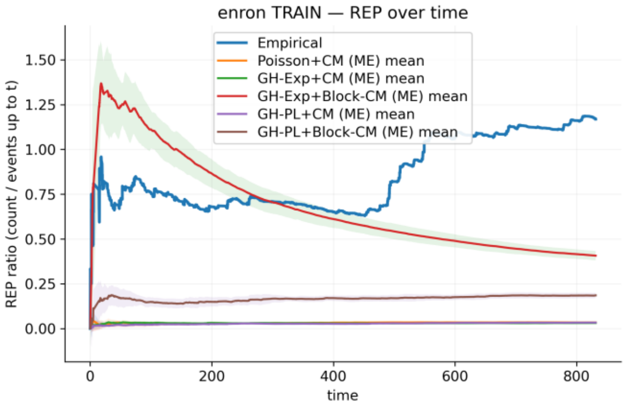

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The future of financial networks

The modeling of the feedback dynamics between the networks and the economic agents unlocks the understanding of the evolution of the variables associated with interconnected economic agents.

Future work for financial network modeling where we can (machine)-learn the relevant -agent and network- features for predicting the evolution of both the agents' variables and the network connections.

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Seabrook, Isobel E. and Fabio Caccioli. "Modeling equity transaction networks as bursty processes" To appear on Data-Driven Modeling

Seabrook, Isobel E. and Fabio Caccioli. "Evaluating structural edge importance in temporal networks." EPJ Data Science 10, no. 1 (2021): 23.

Zhu, Yuwei "Parsimonious Hawkes Processes for temporal networks modelling" To be submitted to CS proceedings

PB "Maximum entropy temporal networks" Physical Review E (2026) Temporal Networks GitHub Code

Questions, comments or ideas? Contact me at p.barucca (at) ucl.ac.uk

Additional Materials

Pattern recognition of financial institutions’ payment behavior

Main findings:

- We present a general supervised machine-learning methodology to represent the payment behavior of financial institutions starting from a database of transactions in the Colombian large-value payment system.
- The methodology learns a feedforward artificial neural network parameterization to represent the payment patterns through 113 features corresponding to financial institutions’ contribution to payments, funding habits, payment timing, payment concentration, centrality in the payment network, and systemic effects due to failure to pay.

Reference

León, Carlos, Paolo Barucca, Oscar Acero, Gerardo Gage, and Fabio Ortega. "Pattern recognition of financial institutions’ payment behavior." *Latin American Journal of Central Banking* 1, no. 1-4 (2020): 100011.

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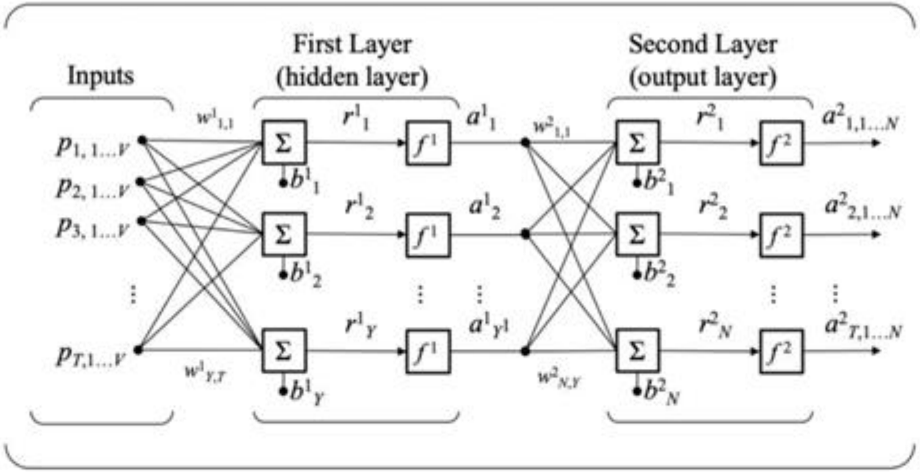
Pattern recognition of financial institutions’ payment behavior

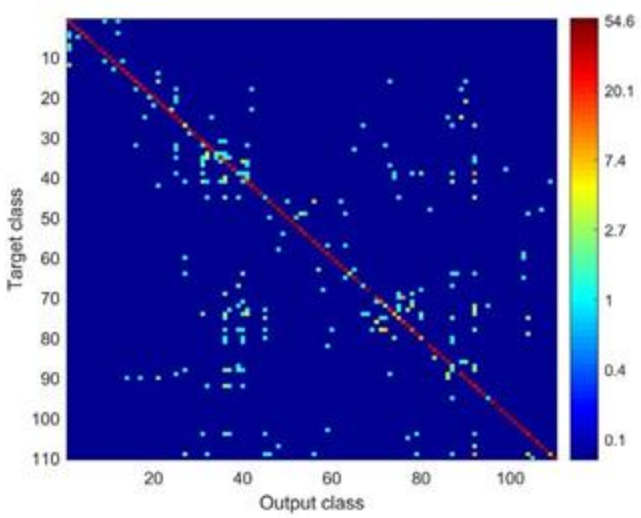


Multi-layer perceptron architecture for classifying institutions.

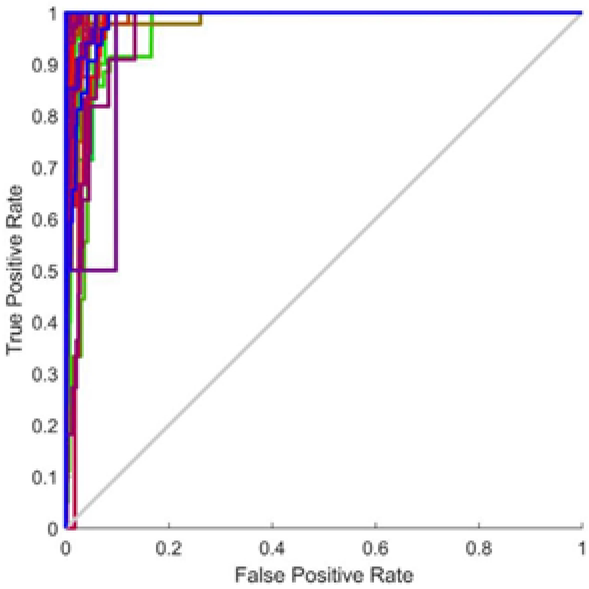
Reference

León, Carlos, Paolo Barucca, Oscar Acero, Gerardo Gage, and Fabio Ortega. "Pattern recognition of financial institutions’ payment behavior." *Latin American Journal of Central Banking* 1, no. 1-4 (2020): 100011.

Pattern recognition of financial institutions' payment behavior



Confusion matrix of the lowest classification errors, including all financial institutions.



ROC curve of the lowest classification error, including all financial institutions.

Reference

León, Carlos, Paolo Barucca, Oscar Acero, Gerardo Gage, and Fabio Ortega. "Pattern recognition of financial institutions' payment behavior." *Latin American Journal of Central Banking* 1, no. 1-4 (2020): 100011.

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